The Knot System

A Numeric Notation of Relationship

by

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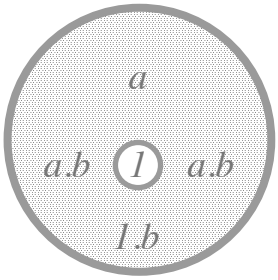

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The Knot System: A Numeric Notation of Relationship

The Knot System is a numerical notation that defines consanguinity. It describes any kind of kinship, simple or complex, that is based on bisexual reproduction. It is exact. Thus, scientifically correct additive relationships – and, by extension, inbreeding coefficients – can be calculated directly. It offers a new, structured way of imagining and visualizing relationships in the human mind, as well as a more precise means of documenting these on paper and in computer output.

By Knud Højrup*

For centuries, humans have sought methods to describe relationships. The most common practice is to assign names to the kinships, such as father, mother, sister, brother, uncle, niece, and grandfather. However, these terms do not define consanguinity precisely enough for maximum scientific use. Geneticists and genealogists need a system that permits them to comprehend more complex relations than those traditionally possible. Sir Francis Galton (1822-1911) stated the problem over a century ago in a letter to the editor of *Nature,* concerning an "Arithmetic Notation of Kinship." According to Galton:

> "Many writers have endeavoured to devise a simple method of describing the various forms of kinship, which, when expressed verbally, are cumbrous and puzzling in the highest degree. I suspect, however, that if we had always been as familiar with the binary system of arithmetic as we are with the decimal, that the facilities afforded by a numerical system of notation of kinship would have been so obvious that it would have been adopted as a matter of course." [1]

Galton's letter briefly discusses the sequential numbering scheme now known by European genealogists as the Kekule von Stradonitz System and by North American genealogists as the Stradonitz or Ahnentafel System. However, the plan was invented by neither Galton nor Kekule von Stradonitz. The latter's name is attached

* Godthåbsvej 14, Astrup; DK-9800 Hjørring, Denmark (Updated 1999). Mr. Højrup is a computer analyst. Preliminary versions of this article were published by him in 1987 and 1988 in Personalhistorisk Tidsskrift, the biannual magazine of the Samfundet for Dansk Genealogi og Personalhistorie (Society for Danish Genealogy and Personal History), which was founded in 1879 as the first of its kind in Nordic countries. Mr. Højrup has been a member of this society since 1981. Figures 1-4 are copies of output from a computer program designed and copyrighted by the author in 1992.

1. Francis Galton, "Arithmetic Notation of Kinship" *Nature* 28(6September 1883):435.

to it because he was the first to describe its properties (in 1898)[2]. The first printed version of this arrangement dates back to 1590, when an Austrian historian and diplomat, Michael Eyzinger, used it in his book on European princely and royal houses.[3] The scheme is so simple and logical that many genealogists since then have used it without knowledge of predecessors and even without considering it in need of explanation. In this article, the method is referred to as the Sequential System.

THE SEQUENTIAL SYSTEM

Widely accepted as the preferred notation for documenting and exchanging genealogical research, its plan is simple. Consecutive integers (whole numbers) are assigned to all ancestors of a person – 1 to the person, 2 to the father, 3 to the mother, 4 to the father's father, etc. Because each person has a countable number of ancestors and there exists an infinite number of integers, it is possible to assign a correct number to any ancestral position, even if other positions remain unidentified. Because of the inherent binary structure of bisexual reproduction, many properties may be derived directly from this numbering plan. Among these:

Sex: Male ancestral positions have even numbers; female ancestors have odd numbers.

Parents: The father of any ancestor n has number $2n$, and the mother has the number $2n + 1$.

Offspring: The offspring of any ancestor n can be calculated as the integer part of $n \div 2$.

Mate: A male ancestor's mate has number $n + 1$, and a female ancestor's mate has number $n - 1$.

Relationship: The exact relationship between any ancestor (n) and the individual at position 1 is found by successively dividing n by 2, discarding fractions at each stage, until reaching the number 1. The resulting list of integers identifies the ancestral positions that form the lineage. The number of times that n is halved equals the number of generations between the individual and the ancestor at position n.

Ancestors per generation: The first ancestral number in every generation (1, 2, 4, 8, 16, etc.) corresponds to the number of ancestor positions in that generation.

Generation numbers: The above numbers are also exponentiations of 2 (i.e., 2^0, 2^1, 2^2, 2^3, 2^4, etc.), and the exponent may be used as the generation number (i.e., 2^4 – or, 2 to the fourth power – represents the fourth ancestral generation).

Although there have been numerous attempts over the years to find an acceptable numbering method that would describe an individual's descendants as well as the Sequential System does for ancestors, no solution has achieved general acceptance. This is also true for the third and most complex kind of relationship – namely, descendants of the ancestors.

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2. Stephan Kekule von Stradonitz, "Über ein zweckmässige Beziffrung der Ahnen," *Vierteljahrsschrift für Wappen-, Siegel-, und Familienkunde* 6(1898),64-72.

3. Michael Eyzinger, *Thesaurus principum hac aetate in Europa viventium* (Cologne: Gottfried von Kempen, 1590)

THE KNOT SYSTEM – DEFINED

Based on the Sequential System, the Knot System recognizes the three basic ways in which individual A may be blood related to individual B:

1. A is ancestor to B.
2. A is descendant of B.
3. A and B have a common ancestor, C,

Beyond this, the Knot System is built upon the following definitions:

Proband
A proband is the individual for whom relationships are to be calculated.

Kin group
The kin group is composed of all individuals related to the proband. It may be reduced in size by specifying a maximum number of generations to ascend from the proband, plus a maximum number of generations to descend from the ancestor. Or the kin group could be confined to all relatives within a total number of generations from the proband.

Ascent list n
An ascent list identifies all individuals who link a person to his or her ancestor at position *n*. The person and the ancestor are included. Ascent List 1 consists only of the person.

Knot individual
A knot individual is an ancestor common to both the proband and the related person. The knot individual must be the only individual appearing on both of the ascent lists that link the three persons together.

Consanguinity element
A consanguinity element is expressed as *a.b* – that is, *a* represents the ascent list that links the proband to the knot individual; and *b* denotes the ascent list that links the related person to the same individual.

This consanguinity element represents an exact description of any one relationship between the proband and the relative. The number of generations between the proband and the relative equals the sum of the number of generations expressed by each of the two ascent lists.

KinCode
The KinCode of a related individual contains all known consanguinity elements that link the individual to the proband. The consanguinity elements must be arranged in numerical ascending order within the KinCode, and every element must be different from all other elements.

Primary and secondary KinCode

The *primary* KinCode always contains the consanguinity element with the smallest number of generations between the proband and the related individual. In a case of more than one element with the same lowest number of generations, the numerically smaller element is selected, together with its possible mate element. (A mate element is one in which the numbers on each side of the decimal point are odd numbers, each being one number higher than the corresponding part of the basic element.) An element pair is preferred over a single element, even when the single element is numerically smaller than the first of the pair.

The remaining consanguinity elements of the relative's KinCode constitute the *secondary* KinCode.

Kin register

A kin register is a list of all individuals of a kin group. Its main sorting criterion is the individual's primary KinCode.

THE KNOT SYSTEM – APPLIED

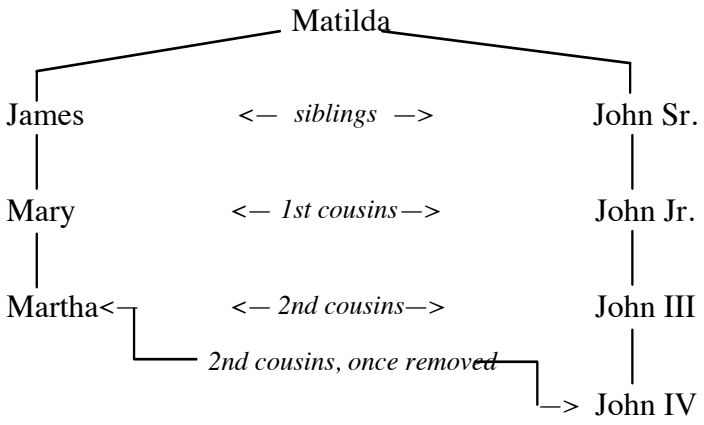
The list of individuals in Ascent List *N* is calculated by successively dividing *n* by 2 and discarding all fractions until 1 is reached. The list includes both the person (called Person A in this example) and the ancestor (*n*). Thus, Person A's Ascent List 13 is composed of the following:

Ancestor 13 (whose number is halved to create 6.5; the fraction is then dropped)
Ancestor 6 (whose number is halved to create 3)
Ancestor 3 (whose number is halved to create 1.5; the fraction is then dropped)
Ancestor 1

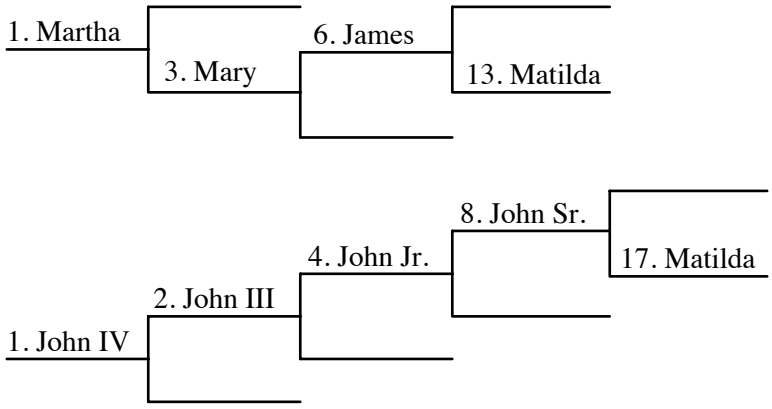
This means that A's Ascent List 13 consists of four individuals: A himself or herself (*n*l), A's mother (*n*3), the mother's father (*n*6), and the mother's father's mother (*n*13). A's gender is unspecified. For the remaining individuals on the list, the gender is defined by the ancestor's position number as previously stated: even numbers represent males, odd, females. Therefore, A's Ascent List *N* is an exact description of A's kinship to the ancestor at position *n*.

The most complex of the three basic relationships set forth on the preceding page – i.e., the relationship between A and B, who have common ancestor C – can now be simply defined. One ascent list is compiled for A, beginning with him or her and ending with C. A second ascent list is compiled for B, beginning with him or her and ending with C. The resulting list number for A is linked to the resulting list number for B, by use of a decimal point – *a.b* being the manner of expressing this basic relationship.

To illustrate, consider the relationship that is verbally identified as *second cousins, once removed*. It is typically depicted on a drop chart in the following manner:



By comparison, separate ancestor charts for Martha and John would express the above lines as follows:



Martha and John are obviously related through Matilda, who is the knot individual because she ties a "knot" of relationship between Martha and John IV. Under the Knot System, Martha's direct line that is depicted above would be identified as her Ascent List 13. John IV's line would be his Ascent List 17.

These two ascent lists are combined, with a decimal point as separation, to form the consanguinity element 13.17 – a combination that looks like an ordinary number (an integer + fraction) but is not. Recalling that the form for expressing the consanguinity element is *a.b,* one could verbally read this consanguinity element 13.17 as: "The ancestor of the proband, Martha, at position *a* (13) is the same individual as the ancestor of her relative, John IV, at position *b* (17)."

It is obvious that all ancestors to C are also common ancestors to A and B. However, only C needs to be considered when numerically describing the basic relationship between A and B, because C is the common ancestor closest to the proband A and the related individual B.

For all practical purposes, the consanguinity elements can be treated as real numbers capable of being compared and sorted – as long as one precaution is observed. For all consanguinity elements that have identical numbers to the left of the decimal point, the expressions to the right of the decimal point should all have the same number of digits. (Zeroes can be added to the right hand number to achieve this, but any added zero must appear immediately after the decimal point. Table 1 illustrates the point, to use consanguinity elements 46.068 and 46.092 as examples: zeroes were added to each so that 68 and 92 would have the same number of digits as 116.) Experience has proved it practical, always, to have at least two digits to the

right of the decimal point. This permits the ascent list to span more than five generations and accommodate most known descendant relations.

Complex relationships

One consanguinity element is an exact description of a relationship when there exists only one knot individual between the proband and the relative. However, this is not always the case. For instance, full siblings of the proband will have both the father and the mother as common ancestors, two situations that fulfill the knot individual definition. Thus, in order to describe exactly a full sibling's relationship to the proband, two consanguinity elements are needed one with the father as the knot individual (2.02), and one with the mother as the knot individual (3.03). The second element (3.03), is considered the mate element to the first, following the definition offered on page 118.

In some instances, there may be several knot individuals linking a proband with another specific relative. A knot individual can even appear in more than one ancestral position on both the proband's pedigree chart and that of the relative. In order to describe fully the kinship in these cases, a separate consanguinity element is required for each of this ancestor's positions and all combinations of them. The knot individual definition though, must be fulfilled for all resulting elements. The related individual's KinCode will include all known consanguinity elements.

In human genealogy, a verbal notation such as brother, sister, cousin, uncle, etc., is used to describe the closest known relationship between two persons. The words of this notation are often accompanied by either "full" or "half" to indicate whether the common ancestry comprises a couple or a single person. This closest relationship is the one the Knot System expresses as the primary KinCode.

A complex kinship is illustrated in tables 1 through 3, which treat the connection of Queen Elizabeth II of Great Britain and Northern Ireland to King Harald of Norway. The queen is the proband. Her kin group is restricted to a five-generation ascent and a six-generation descent. Within this scope, there are fourteen consanguinity elements that specify exactly the relationship between Queen Elizabeth and King Harald. The knot individual, Prince Friedrich von Hessen-Kassel, appears only once in Queen Elizabeth's pedigree (at position 46), but three times in King Harald's pedigree (at positions 68, 92, and 116). Thus, there are three corresponding consanguinity elements in King Harald's KinCode. Table 2 expands one of these elements, 46.068, to show all individuals constituting that basic relation between these royal monarchs.

Table 1

KinCode for Monarchs of Great Britain and Norway

Proband:	Elisabeth II, Queen of Great Britain and Northern Ireland
Relative:	Harald, King of Norway
Scope:	Ascend 5 generations; descend 6 generations

KINCODE	KNOT-INDIVIDUAL
Primary:	
8.10	Edward VII, King of Great Britain and Ireland
9.11	Alexandra, Queen of Great Britain and Ireland
Secondary:	
18.16	Christian IX, King of Denmark
18.28	Same as 18.16
19.17	Louise, Queen of Denmark
19.29	Same as 19.17
44.84	George III, King of Great Britain and Ireland
45.85	Charlotte, Queen of Great Britain and Ireland
46.068	Friedrich, Prinz von Hessen-Cassel
46.092	Same as 46.068
46.116	Same as 46.068
47.069	Caroline, Prinzessin von Nassau-Usingen
47.093	Same as 47.069
47.117	Same as 47.069
Total:	14 consanguinity elements

A different representation of the same relationship:

Primary KinCode:	8.10 9.11
Secondary KinCode:	18.16 18.28 19.17 19.29 44.84 45.85 46.068 46.092 46.116 47.069 47.093 47.117

Table 2

Expanded View of Consanguinity Element 46.068

Proband:
1 Elizabeth II, Queen of Great Britain and Northern Ireland
2 George VI, King of Great Britain and Ireland
5 Mary, Queen of Great Britain and Ireland
11 Mary Adelaide, Princess of Great Britain and Ireland
23 Augusta, Prinzessin von Hessen-Kassel
Knot-individual: 46.068 Friedrich, Prinz von Hessen-Kassel
34 Wilhelm, Prinz von Hessen-Kassel
17 Louise, Queen of Denmark
8 Frederik VIII, King of Denmark
4 Haakon VII, King of Norway
2 Olav V, King of Norway
1 Harald, King of Norway
Relative.

Additive relationships

"Additive relationship" is a genetic term used to measure the fraction of "like genes" shared by two humans or animals. It suggests how reliable the records of one will be in predicting genetic matters for the other.[4] Determining the additive relationship between any two individuals is a two step process.

- First, determine the number of generations separating them, by using the mathematical process described on page 116 under "relationship." The resulting number is expressed as *n* in the formula at the bullet below.

  (Alternately, one may simply count the number of generations intervening between the two individuals – ascending from Person A to the common ancestor, C, and descending back down to Person B. Persons A and B, themselves, are not counted. C is counted in both the ascent and the descent. The total number of generations between A and B is the generational *n*.)
- Second, calculate the actual additive relationship by applying the formula $(1/2)^n$ – meaning *one* divided by *two*, raised to the *nth* power.

To continue our example of Queen Elizabeth and King Harald, it is easy to see from table 2 that eleven generations separate them in the basic calculation. If this were their only kinship, then their additive relationship would be 0.00048828125 –

[4]. L. Dale Van Vleck, E. John Pollak, and E. A. Branford Oltenacu, *Genetics for the Animal Sciences* (New York: W. H. Freeman, 1987), 199. The *inbreeding coefficient* is one half the additive relationship between parents.

or, expressed more simply, they would be 0.049% related to each other. This table is arrived at through the following calculation:

$$n = 5 + 6 = 11$$
$$(1/2)^{11} = 0.00048828125$$

In a complex kinship, the total additive relationship is calculated by adding together the individual additive relationships already determined for each consanguinity element. Table 3 lists the additive number for each relationship shown for Queen Elizabeth and King Harald and then calculates the total additive relationship – i.e., 0.05078125, which makes them 5.08% related to each other.

Table 3

Additive Relationships between Monarchs of Great Britain and Norway

Proband: Elisabeth II, Queen of Great Britain and Northern Ireland
Relative: Harald, King of Norway
Based on: Table 1

ADDITIVE RELATIONSHIP*	KNOT-INDIVDUALS FROM WHOM RELATIONSHIPS ARE CALCULATED
Primary:	
0.015625	Edward VII, King of Great Britain and Ireland
0.015625	Alexandra, Queen of Great Britain and Ireland
Secondary:	
0.00390625	Christian IX, King of Denmark
0.00390625	Same as above
0.00390625	Louise, Queen of Denmark
0.00390625	Same as above
0.000488281	George III, King of Great Britain and Ireland
0.000488281	Charlotte, Queen of Great Britain and Ireland
0.000488281	Friedrich, Prinz von Hessen-Cassel
0.000488281	Same as above
0.000488281	Same as above
0.000488281	Caroline, Prinzessin von Nassau-Usingen
0.000488281	Same as above
0.000488281	Same as above
0.05078125	Total for 14 consanguinity elements

** a genetic measure of the fraction of like genes shared by Queen Elizabeth and King Harald*

Table 4

Three Generation Kin Register for Queen Elizabeth II

PRIMARY KINCODE		KIN GROUP MEMBER
1		Elizabeth II, Queen of Great Britain and Northern Ireland
1.03		Anne, Princess of Great Britain and Northern Ireland
1.03		Charles, Prince of Wales
1.03		Andrew, Prince of Great Britain and Northern Ireland
1.03		Edward, Prince of Great Britain and Northern Ireland
1.05		William, Prince of Great Britain and Northern Ireland
1.05		Harry, Prince of Great Britain and Northern Ireland
1.05		Beatrice Mountbatten-Windsor
1.05		Eugenie Mountbatten-Windsor
1.07		Peter Mark Andrew Phillips
1.07		Zara Anne Elizabeth Phillips
2		George VI, King of Great Britain and Ireland
2.02	3.03	Margaret Rose, Princess of Great Britain and Northern Ireland
2.06	3.07	David Armstong-Jones, Viscount Linley
2.06	3.07	Sarah Frances Elizabeth Armstong-Jones, Lady
3		Elizabeth, Queen of Great Britain and Ireland
4		George V, King of Great Britain and Ireland
4.02	5.03	Edward VIII, King of Great Britain and Ireland
4.02	5.03	Mary, Princess of Great Britain and Ireland
4.02	5.03	Henry, Prince of Great Britain and Ireland
4.02	5.03	George, Prince of Great Britain and Ireland
4.02	5.03	John, Prince of Great Britain and Ireland
5		Mary, Queen of Great Britain and Ireland
6		Claude George, 14th Earl of Strathmore and Kinghorne
6.02	7.03	Violet Hyacinth Bowes-Lyon, Lady
6.02	7.03	Mary Frances Bowes-Lyon, Lady
6.02	7.03	Patrick, 15th Earl of Strathmore and Kinghorne
6.02	7.03	John Herbert Bowes-Lyon, Honorable
6.02	7.03	Alexander Francis Bowes-Lyon, Honorable
6.02	7.03	Fergus Bowes-Lyon, Honorable
6.02	7.03	Rose Constance Bowes-Lyon, Lady
6.02	7.03	Michael Claude Hamilton Bowes-Lyon, Honorable
6.02	7.03	David Bowes-Lyon, Honorable Sir
7		Nina Cecilia, Lady of Strathmore and Kinghorne
8		Edward VII, King of Great Britain and Ireland
9		Alexandra, Queen of Great Britain and Ireland
10		Franz, Prince of Württemberg
11		Mary Adelaide, Princess of Great Britain and Ireland
12		Claude, 13th Earl of Strathmore
13		Frances Dora Smith of Blendon Hall
14		Charles William Francis Cavendish-Bentinck
15		Caroline Louise Cavendish-Bentinck

Kin register

If the members of a kin group are sorted in ascending numerical order on their primary KinCode, the resulting sequence shows an interesting pattern. The first individual will be the proband (primary KinCode = 1), followed by the proband's own descendants, generation by generation (see table 4). The descendants are followed by the proband's father and his descendants, one generation after the other. This set is followed by the mother and any descendants she may have by a mate other than the proband's father. This pattern continues through the generations of ancestors and their descendants for all members of the kin group.

This sequence of individuals resembles very much the relationship patterns expressed in ancient laws for inheriting property and titles. When an individual (the proband) dies, the children are the prime heirs. If those children are dead, their descendants, one generation after the other, will inherit. If all descendants of the proband are extinct, the proband's parents and their descendants are next in the line of inheritance. If they are deceased too, the proband's grandparents and their descendants come next in the line – and so on until a proper heir is found.

USES FOR THE KNOT SYSTEM

A scientifically correct notation for all kinds of bisexual relationships has not existed before, to this writer's knowledge. It is therefore difficult to predict all the applications the Knot System may have, but some are already obvious. The system

- offers a modular way to document every known biological relation between two individuals, thereby making it easy to verify that new found relations are not already known.
- facilitates an exchange of research results. It is now possible to describe exactly the relationship between two individuals without the need to carry information on the individuals constituting the relationship.
- can be used to make consanguinity registers, in which individuals within a kin group are organized by their primary relationship to the proband.
- exploits computer capabilities more fully. Because computers can process very large kin groups, it is now possible to have them calculate and present very complex relationships in a humanly comprehensible form.
- is a valuable genetic research tool. If several individuals in a kin group show any interesting trait or disease, visual analysis of their KinCodes might reveal the ancestors from whom the phenomenon could have been inherited.
- is very suitable for teaching simple and complex relationships, because of its clear definitions and its structured, modular design.

CONCLUSION

The Knot System is a numerical notation that defines consanguinity. It describes any kind of kinship, simple or complex, that is based on bisexual reproduction. It is exact. Thus, scientifically correct additive relationships – and, by extension, inbreeding coefficients – can be calculated directly from the KinCode. It offers a new, structured way of imagining and visualizing relationships in the human mind, as well as a more, precise means of documenting these on paper and in computer output.

The Knot System does not seek to replace the established genealogical schemes. Rather, it is an extension of the internationally used, almost natural notation for human ancestors – the Stradonitz or Ahnentafel System. It is language independent, offering a common relationship terminology and notation that works for animals, fish, humans, insects, plants, and other organisms that procreate bisexually. It should enable geneticists and genealogists to comprehend and document more complex relations than ever before, which could have a major impact on research in both fields.

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